

Buoyancy, Stability, and Ballast 1

Cornerstone Electronics Technology and Robotics III

(Notes primarily from “Underwater Robotics – Science Design and Fabrication”, an excellent book for the design, fabrication, and operation of Remotely Operated Vehicles ROVs)

- **Administration:**
 - Prayer
- **Basic Buoyancy Principles:**
 - **Definition:** Buoyancy is the upward force exerted by the fluid on a body that is immersed.
 - **Buoyant force:** When an object displaces water, the water surrounding it has the tendency to try to fill in the space the object now occupies. The water pushes against the object, exerting pressure and force on it. Since pressure increases with depth, the pressure-induced force on the bottom of an object is greater than the pressure-induced force acting on the top of the object. The buoyant force is the difference between the larger force produced from the higher pressure pushing up under the object and the smaller force produced from the lower pressure pushing down over the object. See Figure 1.

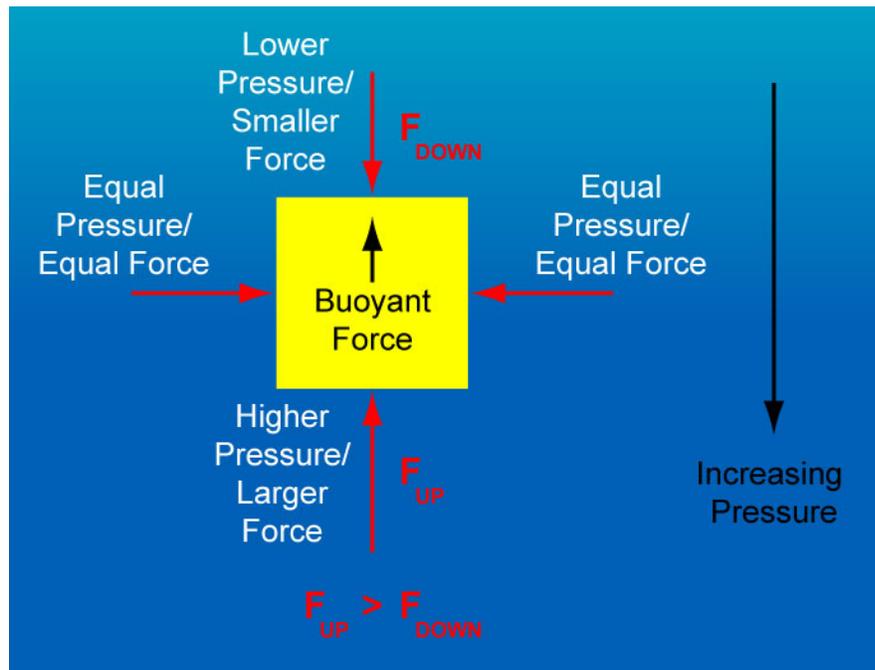


Figure 1: The Net Force of the Fluid Acting on an Object Is the Buoyant Force

- All submerged objects are subject to an upward buoyant force. Even objects that sink undergo this upward force and therefore weigh less in water than in air.

- **Archimedes Principle:** The magnitude of the upward force (buoyant force) of an object immersed in a fluid is equal to the magnitude of the weight of fluid displaced by the object.

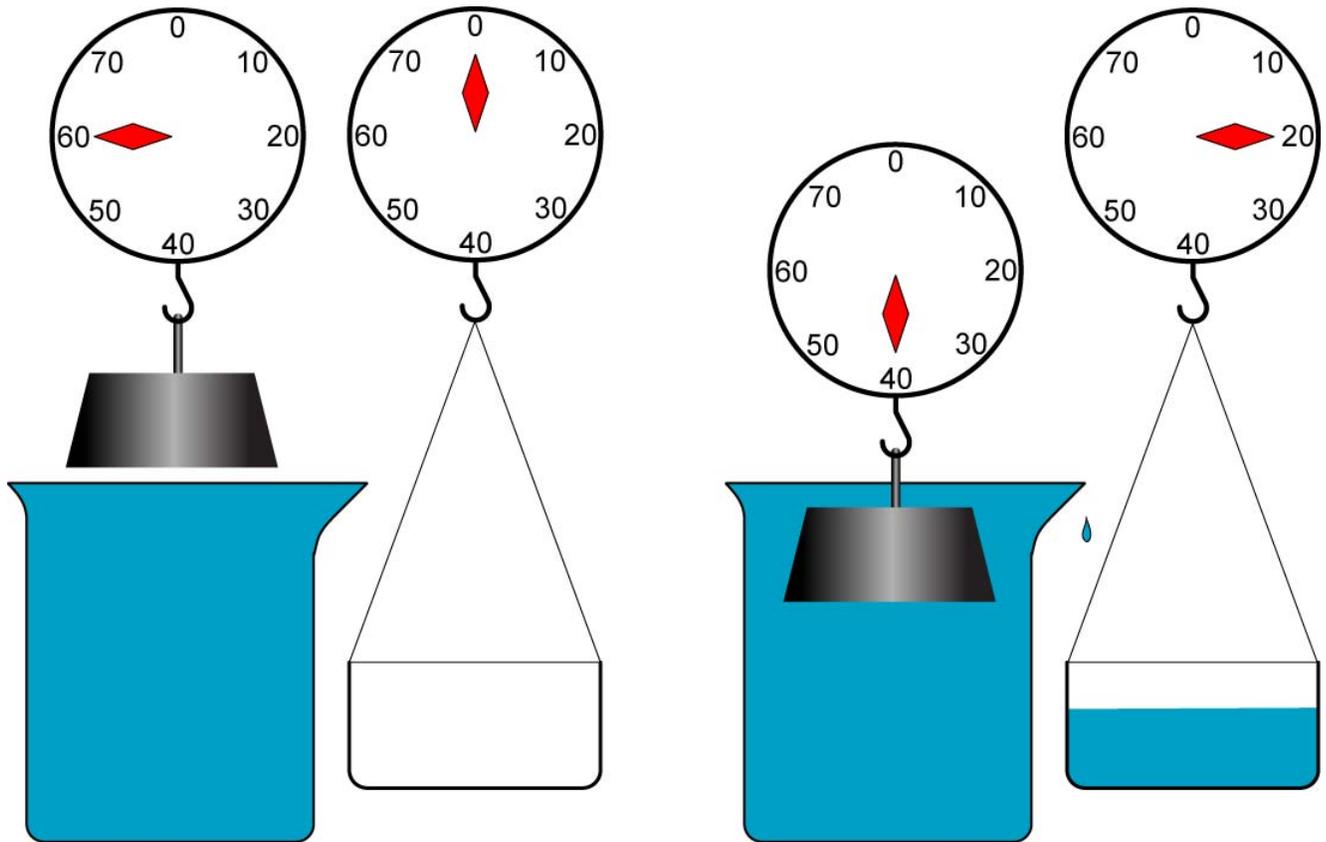


Figure 2: Archimedes Principle – The Buoyant Force is Equal to the Weight of the Displaced Water

- For example, calculate the upward buoyant force caused when completely submerging a closed-cell foam water tube that has a hollow center and is 6.432" long.

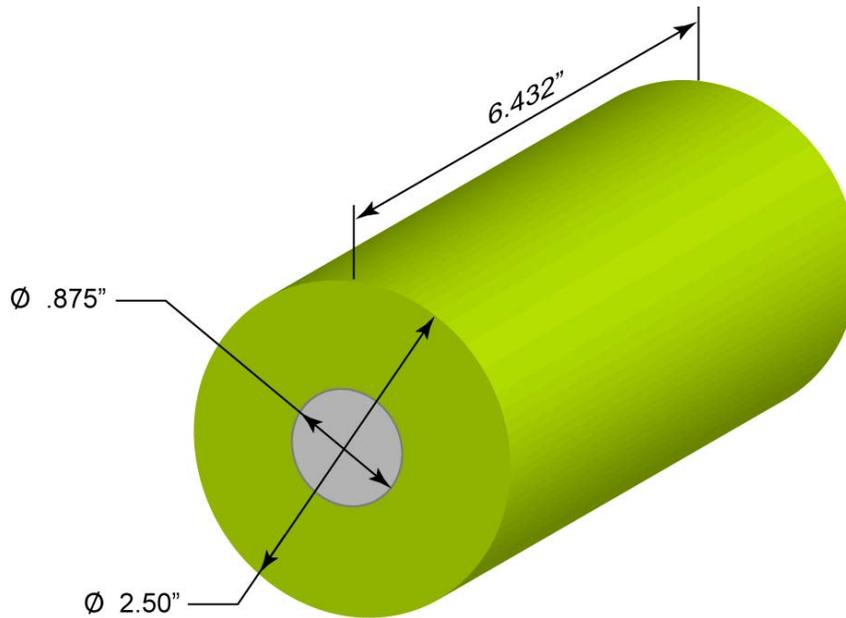


Figure 3: Closed-Cell Foam Water Tube 6.432" Long

Calculate the upward buoyant force:

First, find the displaced volume of the tube:

$$\begin{aligned} \text{Volume} &= (\pi (\text{Outside Rad.})^2 - \pi (\text{Inside Rad.})^2) \times \text{Length} \\ \text{Volume} &= (\pi (1.25 \text{ in})^2 - \pi (0.4375 \text{ in})^2) \times 6.432 \text{ in} \times 1 \text{ ft}^3/1728 \text{ in}^3 \\ \text{Volume} &= 0.016 \text{ ft}^3 \end{aligned}$$

Now calculate the weight of freshwater that would fill that volume:

$$\begin{aligned} \text{Freshwater Weight} &= \text{Volume} \times \text{Density of Freshwater} \\ \text{Freshwater Weight} &= 0.016 \text{ ft}^3 \times 62.4 \text{ lbs/ft}^3 \\ \text{Freshwater Weight} &= 1.0 \text{ lb} \end{aligned}$$

By Archimedes Principle, the magnitude of the buoyant force of the tube immersed in freshwater is equal to the weight of the displaced water by the tube, so the buoyant force is also equal to 1 pound.

$$\text{Buoyant Force} = 1.0 \text{ lb}$$

The weight of the foam tube is approximately 0.015 pounds. The buoyant force is much greater than the weight of the foam tube so the immersed tube will quickly rise to the surface and float.

If the tube were made of [6061 aluminum alloy](#) instead of foam, it would weigh 2.7 lbs ($0.016\text{ft}^3 \times 1728 \text{ in}^3/\text{ft}^3 \times 0.0975 \text{ lb/in}^3$) and it would sink (2.70 lbs. > 1.0 lbs.).

- **Positive, Neutral, and Negative Buoyancy:**
 - A submerged object like a ROV will float or sink depending upon the net effect of the weight of the object and the buoyant force generated by the object.
 - Three possible conditions for the object (see Figure 4):
 - Positive buoyancy: buoyant force > weight, the object floats
 - Neutral buoyancy: buoyant force = weight, the object neither floats or sinks
 - Negative buoyancy: buoyant force < weight, the object sinks

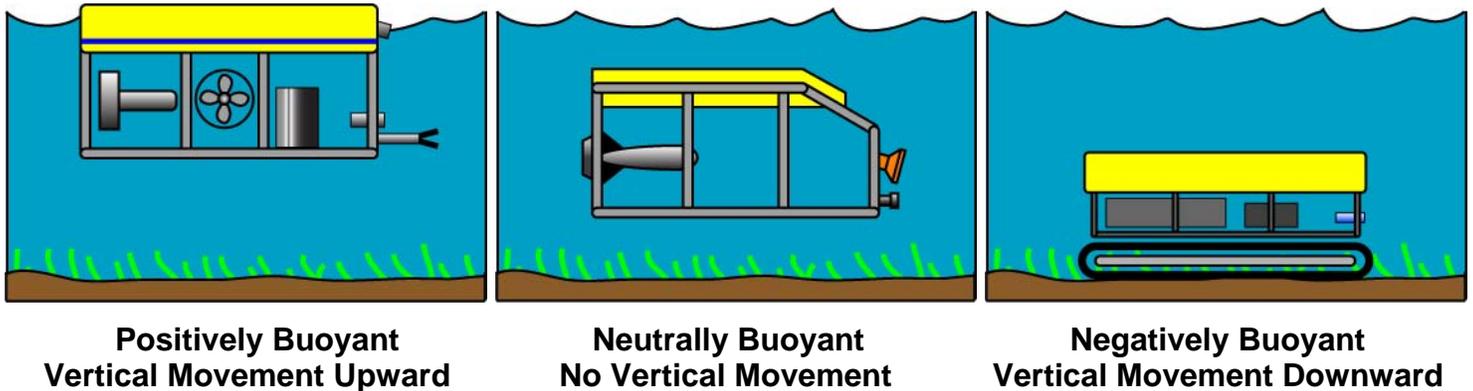


Figure 4: Positive, Neutral, and Negative Buoyancy

- Three ways to predict positive, neutral, and negative buoyancy:
 - By weight: Compare the weight of the object to the weight of the water the object displaces.
 - If the object weight > the weight of the water the object displaces, then the object sinks
 - If the object weight = the weight of the water the object displaces, then the object neither sinks or floats
 - If the object weight < the weight of the water the object displaces, then the object floats
 - By density: Compare the object's density to the density of the water the object is submerged in.
 - If the object's density > the density of water, then the object sinks
 - If the object's density = the density of water, then the object neither sinks or floats
 - If the object's density < the density of water, then the object floats
 - By specific gravity: Compare the object's specific gravity to the specific gravity of the water the object is submerged in.
 - If the object's specific gravity > the specific gravity of the water, then the object sinks
 - If the object's specific gravity = the specific gravity of the water, then the object neither sinks or floats
 - If the object's specific gravity < the specific gravity of the water, then the object floats

- **Designing for Buoyancy:**

- Introduction:

- Various vehicles have distinct buoyancy requirements. For example, boats need to be positively buoyant, bottom-crawling vehicles require negative buoyancy, and submarines and ROVs perform better when they are nearly neutrally buoyant.
 - It is better to design the vehicle for the most favorable buoyancy first rather than building the vehicle and then adjusting the buoyancy by adding weights or floats.
 - The goal for ROVs and AUVs (autonomous underwater vehicles) is for the vehicle to be slightly positive buoyant.
 - Allows the vehicle to float to the surface in the event of propulsion failure
 - Makes modifications easier when the vehicle remains at the surface
 - Easier to add weights than floatation if the vehicle requires trimming
 - As a general rule for small vehicles, the buoyant force should be designed as 4% more than the weight of the vehicle. Weights are then added to bring the buoyant force to within 1 – 2% of the submerged weight.

- Weight Statement Table:

- Weight statement table is a table to organize the data necessary to determine the total buoyancy of a vehicle.
 - The table allows you to update and track your vehicle’s buoyancy as your design progresses.
 - Sample weight statement table:

Project:							
Water Density:							
Last Updated:							
Last Update By:							
Subsystem	Part(s)	Part Weight	Part Shape	Dimensions	Volume of Displaced Water	Weight of Displaced Water	Buoyancy
		(gf)			(cc)	(gf)	(gf)

Figure 5: One Version of a Weight Statement Table

- Objects that come in contact with water add both to the weight and buoyant force. Objects that are enclosed inside a pressure canister only add to the weight and not to the buoyant force.
- Be consistent with the units when entering data. Choose a unit of measure for weight and another for displaced volume and stick with them.
- Perform Buoyancy 1 Lab 1 – Weight Statement Table
- Adjusting Buoyancy:
 - If a vehicle is too positively buoyant, try to add weights inside a pressure canister because they do not displace any additional water thus they do not add any more buoyant force to the vehicle.
 - It is a good idea to have a bolt (trim post) for washers at each corner of the vehicle for easy last minute trimming. Trimming is distributing the load on a vehicle so that it sits well in the water. See Figure 6 below.



Figure 6: Trim Post with Stainless Steel Washer Weights

- **Why Things Tip or Flip Over in Water:**
 - To avoid tipping the vehicle while in water, the vehicle's weight and floatation must be placed properly.
 - Center of Gravity (CG):
 - The single point at which the entire weight of a rigid body may be considered as concentrated. May be thought as the sum of all the gravitational forces acting on a body.
 - Applying the principle of center of gravity allows for the use of simplified equations when studying the effects of weight on a vehicle.
 - The center of gravity of common geometric shapes with uniform mass:

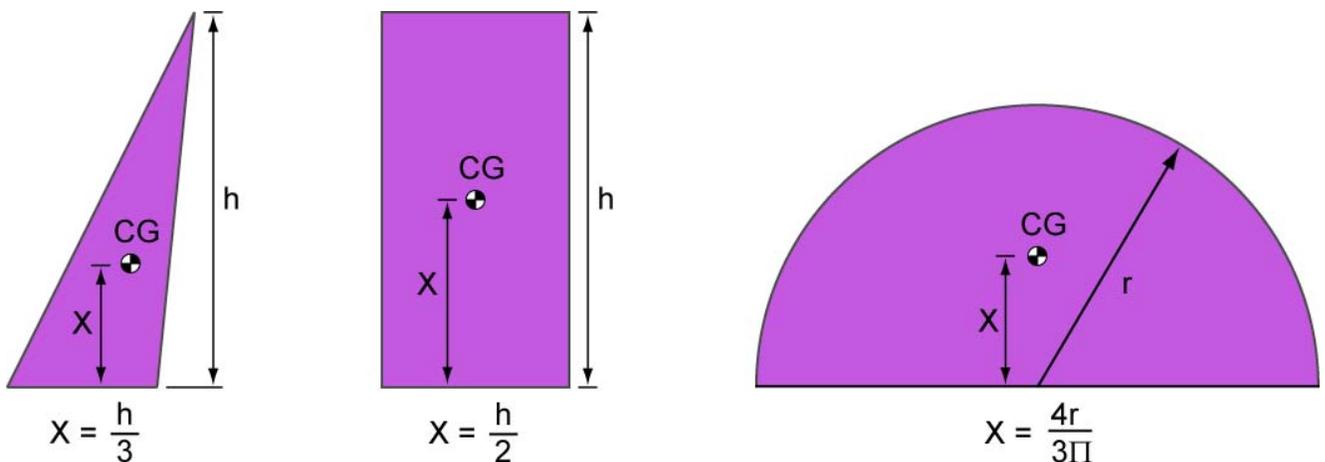


Figure 7: Center of Gravity Equations for Common Geometric Shapes

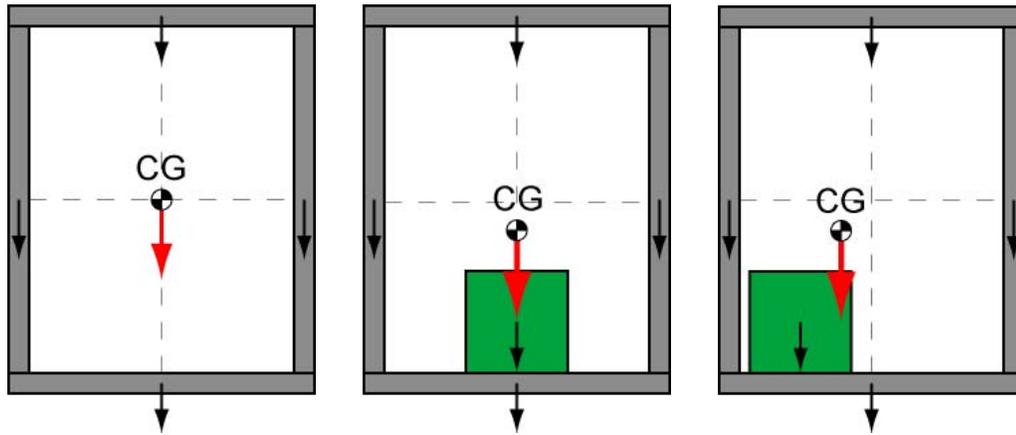


Figure 8: Changes in the Center of Gravity with Changes in Weight Distribution

- Finding the center of gravity empirically using a plumb line:
 - When an object is suspended so that it can move freely, its center of gravity is always directly below the point of suspension.
 - You can empirically find the center of gravity of an object by hanging it from two different points and using a plumb line to mark the vertical lines. The intersection of the two vertical lines from the plumb line is the center of gravity for the object. See Figure 9. This procedure is relatively easy for a flat object. However, it would be more difficult if the object has a three dimensional shape.

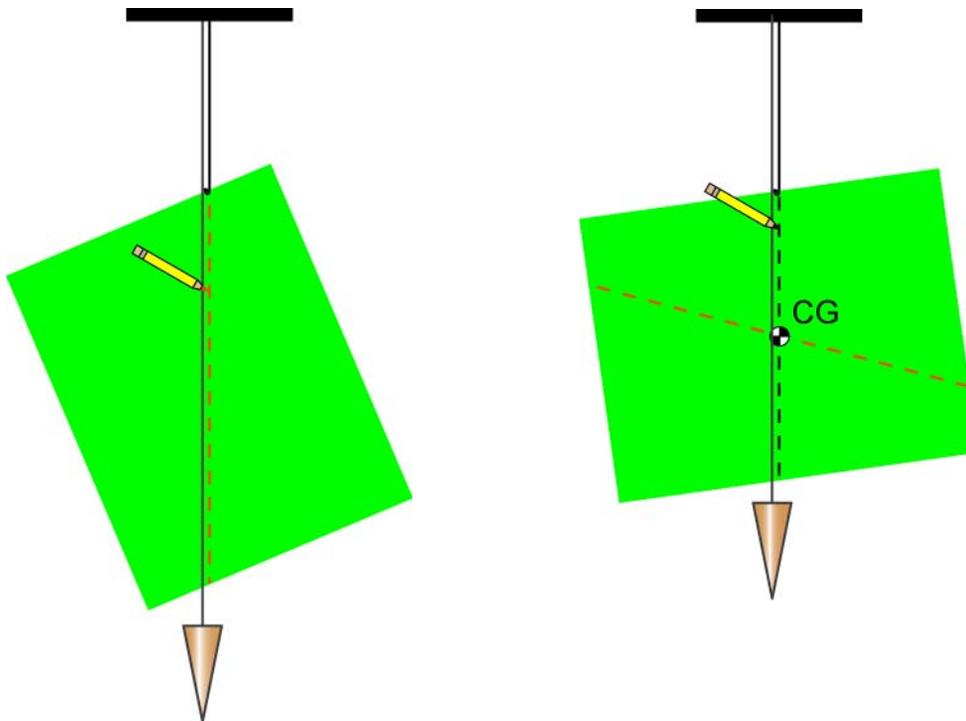


Figure 9: Locating the Center of Gravity Using a Plumb Line

- Likewise, if you balance an object on one of its points, the center of gravity will lie on the normal line passing through the point. See Figure 10.

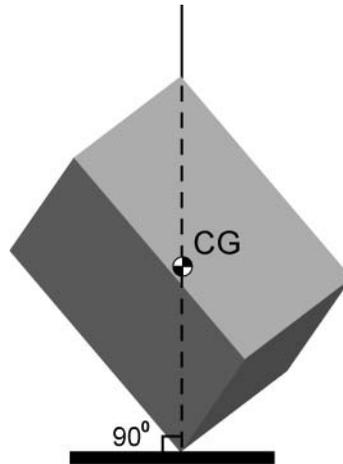


Figure 10: With an Object Balanced on Its Point, the CG Lies on the Line Normal to the Surface

- A lower center of gravity helps prevent a surface vehicle – such as an automobile – from tipping over. Objects will spin about their center of gravity. Also, the center of gravity provides the greatest impact in a collision.
- Center of Buoyancy (CB):
 - When an object is partly or completely submerged in a liquid, some of that liquid is displaced. The volume of displaced liquid is exactly the same as the volume of whatever part of the object is below the surface. The shape of the volume of displaced liquid is also exactly the same as the shape of the part that is submerged, because the object itself is displacing the liquid. The *center of buoyancy* of the object is located at the center of gravity of the volume of the displaced liquid. It is the point through which the upward buoyant force seems to act. See Figures 11a and 11b.

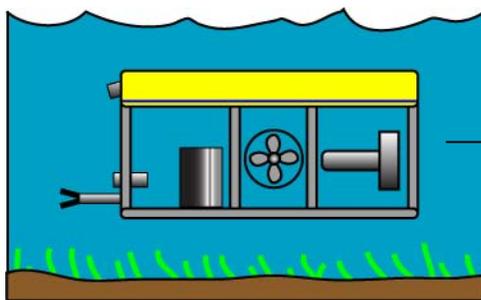


Figure 11a: The Vehicle Fully Submerged

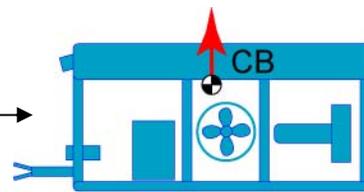


Figure 11b: The Shape of the Displaced Water is Exactly the Same as the Shape of the Vehicle. The Center of Buoyancy is Located at the Center of Gravity of the Volume of Displaced Water

- Torque: A force that tends to rotate or turn things.
 - More specifically, torque τ about a point (p) is equal to the force (F) times the distance measured perpendicular to the line of the force. See Figure 12.

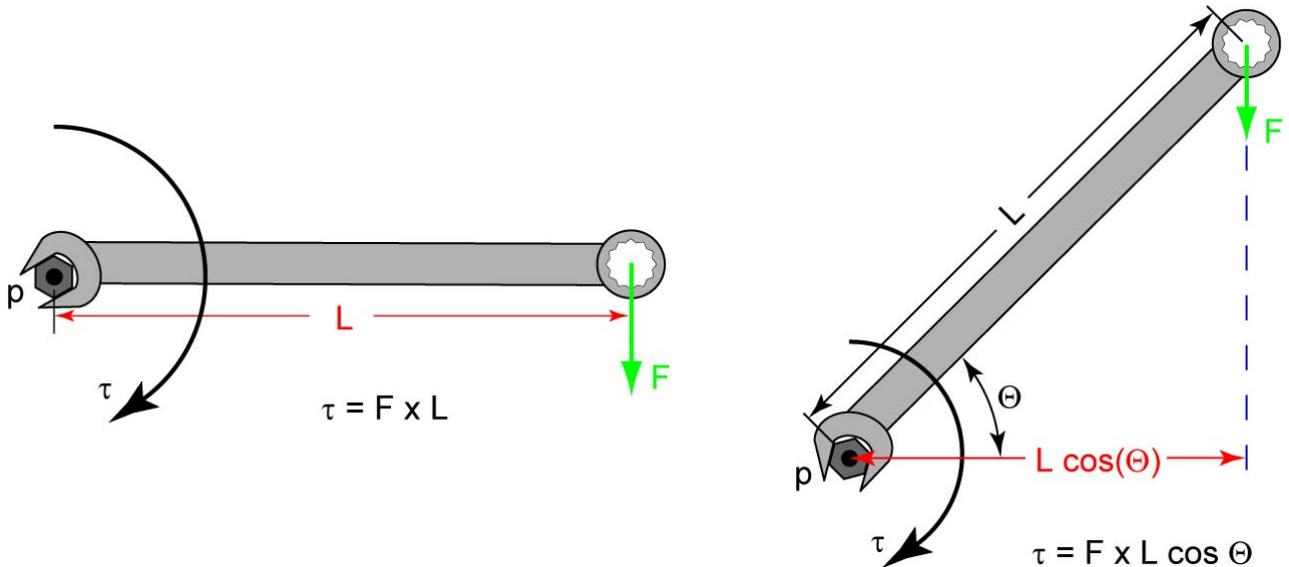


Figure 12: Two Illustrations of Torque τ about Point p Created by Force F

- Torque is measured in units of force times length such as newton-meters (metric) or foot-pounds (imperial).
- Example 1: Calculate the torque in Figure 13 below.

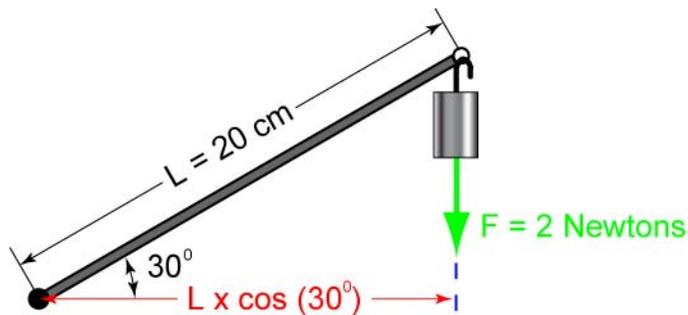


Figure 13: Torque Problem

$$\begin{aligned} \tau &= F \times L \times \cos(30 \text{ degrees}) \\ \tau &= 2\text{N} \times 0.20\text{m} \times 0.866 \\ \tau &= 0.37\text{Nm} \end{aligned}$$

- Equal torques acting on a meter stick:
 - The following system is said to be in equilibrium or balanced if the opposing torques are equal but working in the opposite directions.

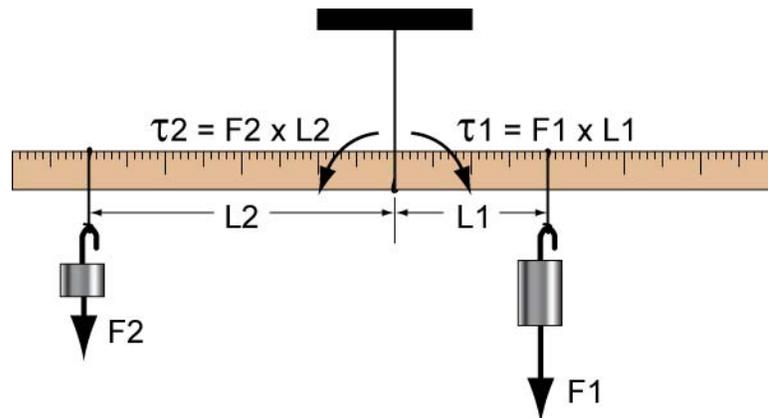


Figure 14: Meter Stick in Equilibrium if $\tau_1 = \tau_2$

- If the system were in equilibrium, the torques would be equal and no net torque would act on the system.
- If the system was not in equilibrium, or unbalanced, the bar rotates about the center pivot string line in the direction of the higher torque.
- Perform Buoyancy 1 Lab 2 – Torque
 - Couples: A consists of **two equal and opposite forces** acting upon a body whose **parallel lines of force** do not coincide (not collinear). The only effect of a couple is to produce or prevent the turning of a body. The moment of the couple (or torque) is measured by the product of the magnitude of either **force** and the **perpendicular distance** between the action lines of the forces. Refer to Figure 15.

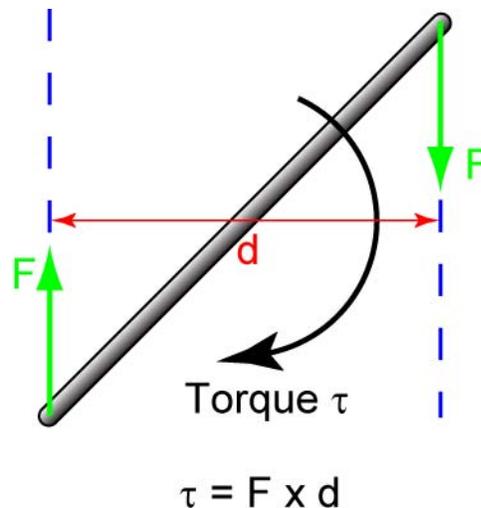


Figure 15: A Couple – Two Equal and Opposite Forces Acting on a Body whose Parallel Lines of Force Do Not Coincide

- Hydrostatic equilibrium:
 - Remember that the goal for ROVs and AUVs is for the vehicle to be only slightly positive buoyant. Let's assume that the ROV is neutrally buoyant and that the weight and buoyant forces are equal, but in opposite directions.
 - A ROV is in equilibrium when the weight and buoyant forces are equal and collinear. See Figure 16.

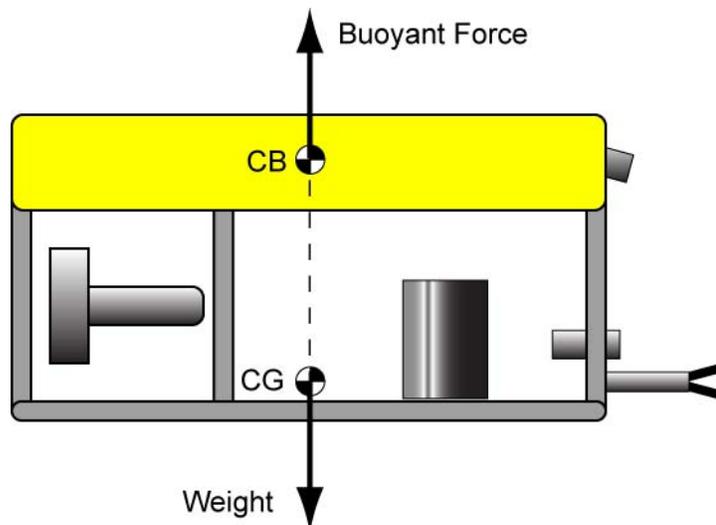


Figure 16: ROV in Static Equilibrium – Weight and Buoyant Forces are Equal and Collinear

- In the case of a submerged ROV that is not in equilibrium, the weight and buoyant force are on parallel lines of force that do not coincide, forming a couple. The couple creates a righting torque (or righting moment) which will bring the ROV back into equilibrium. This is discussed in more detail in Lesson 10.

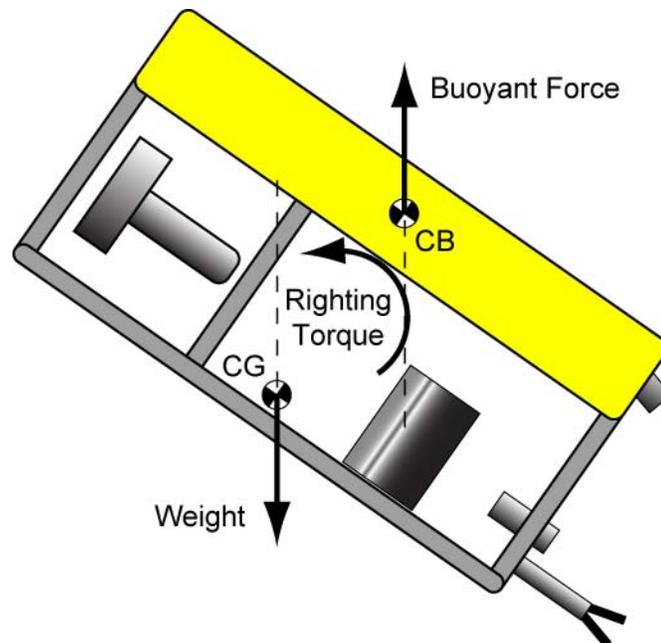


Figure 17: Righting Torque Acting on a ROV which Is Out of Equilibrium

- Place floating components on the top part of your vehicle and heavy sinking parts near the bottom of your vehicle. This ensures that the righting torque will work properly. If you do not do this, the CB is below the CG and the vehicle will be upside down when reaching equilibrium. See Figure 18.

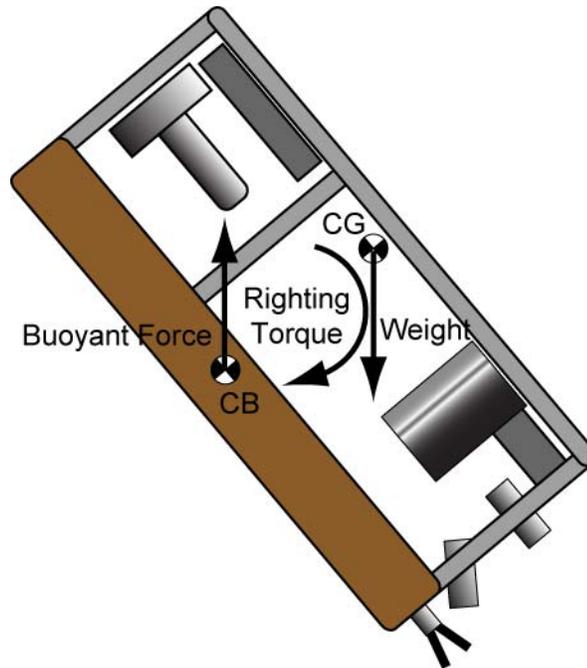


Figure 18: ROV Flipping Upside Down because the CB is below the CG

- Misplacing the floating and heavier ROV parts creates equilibrium positions that may be impractical and unworkable. See Figure 19.

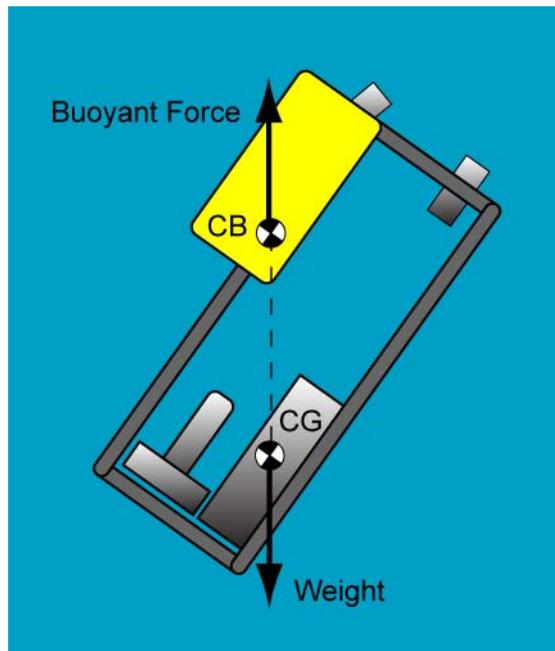


Figure 19: ROV in Equilibrium with Misplaced Center of Gravity and Center of Buoyancy

Cornerstone Electronics Technology and Robotics III Buoyancy, Stability, and Ballast 1 Lab 1 – Weight Statement Table

- **Purpose:** Students use a canister they built to complete a weight statement table.
- **Apparatus and Materials:**
 - 1 – Plexiglas Pressure Canister from Lesson 8 Lab
 - For Lesson 8 as a .pdf file, see:
http://cornerstonerobotics.org/curriculum/lessons_year3/eriii8_pressure_hulls_canisters3_build_canister.pdf
- **Procedure:**
 - Download the .pdf or .xls Excel version of a blank weight statement table.
 - For the .pdf version, link to:
http://cornerstonerobotics.org/excel%20doc/year3/buoyancy1_weight_statement_table.pdf
 - For the .xls version, link to:
http://cornerstonerobotics.org/excel%20doc/year3/buoyancy1_weight_statement_table.xls
 - A discussion about table entries is on the next page.

Project:
Water Density:
Last Updated:
Last Update By:

Subsystem	Part(s)	Weight (gram force) (gf)	Part Shape	Dimensions	Volume of Displaced Water (cubic cm) (cc)	Weight of Displaced Water (Buoyancy Force) (gf)	Net Buoyancy (Buoyancy Force - Weight) (gf)
Totals:	—		—	—			

- Entering information and data in the table (see the textbook for a more complete treatment of each entry):
 - Title Information: Keep the information in the title block up to date. It is advisable to encode each update as a new file.
 - Subsystem column: Group all the parts that are included in a subsystem in one section of your table.
 - Part column: Include a brief description of each part.
 - Part Shape column: Contains a word or two explaining the shape of the object.
 - Dimensions column: Enter the vital dimensions used for calculating the displaced volume.
 - Volume of Displaced Water column: When possible, calculate the displaced volume for each part using formulas for the appropriate geometric shape. If the part's shape is irregular, it makes the volume calculations difficult. You can submerge a small part in a graduated cylinder to measure its displaced volume. Remember, 1 ml = 1 cc. If the part is large, use a bucket that is completely full to the rim and collect and measure all of water that overflows when the part is carefully submerged.
 - Weight of Displaced Water column: If the part does not touch water, enter zero for that part. If the part makes contact with water, multiply the water density times the Volume of Displaced Water entry. The weight of the displaced water is equal to the buoyancy force acting on the immersed object.
 - Net Buoyancy column: Subtract the Part Weight from the Weight of Displaced Water. If the net buoyancy is positive, the part will float; if negative, the part will sink.
 - Totals row: Total weight will tell you how much your subsystem weighs out of the water. Total weight of displaced water specifies the total buoyancy force. Total net buoyancy indicates whether the subsystem will sink (- value) or float (+ value).

- **Results:**
 - Your results should be fairly close to the following table.

Project: Year 3 Canister							
Water Density: 1.0000 gm/cc							
Last Updated: JLK							
Last Update By: 9/22/12							
Subsystem	Part(s)	Weight (gram force) (gf)	Part Shape	Dimensions	Volume of Displaced Water (cubic cm) (cc)	Weight of Displaced Water (Buoyancy Force) (gf)	Net Buoyancy (Buoyancy Force - Weight) (gf)
Canister	8 - 1/4 Washers	11	Washers	Measured in Water	2	2	-9
Canister	8 - 1/4 Nuts	25	Nuts	Measured in Water	4	4	-21
Canister	4 - 1/4x20 All-Thread	166	All-Thread	23 cm Long	24	24	-142
Canister	Aluminum End Cap	168	Cuboid	10.2 x 10.2 x 0.6	62.4	62	-106
Canister	Plexiglas End Plate	160	Cuboid	10.2 x 10.2 x 1.3	135.3	135	-25
Canister	Plexiglas Cylinder	148	Hollow Cylinder	7.6 OD 7.0 ID x 18.1	125	125	-23
Canister	Air in Cylinder	0	Cylinder	7.0 ID x 18.1	697	697	697
Canister	RubberGasket	14	Cuboid	10.2 x 10.2 x 0.12	12	12	-2
Canister	Total Buoyancy	692	-	-	-	1061	369

Plexiglass Cylinder: $Vol = (\pi(OD/2 * OD/2) - \pi(ID/2 * ID/2)) * Length$
 $Vol = ((3.8*3.8*3.1416)-(3.5*3.5*3.1416))*18.1$
 $Vol = 125 \text{ cc}$

Air in Cylinder: $Vol = \pi(ID/2 * ID/2) * Length$
 $Vol = (3.5*3.5*3.1416)*18.1$
 $Vol = 697 \text{ cc}$

- **Conclusions:**
 - Will the pressure canister float or sink?
 - To make the buoyant force for the canister 4% more than the weight, how many gf of weight or floatation will you have to add to the canister?

Cornerstone Electronics Technology and Robotics III

Buoyancy, Stability, and Ballast 1 Lab 2 – Torque

- **Purpose:** The student explores the concepts of torque and calculates the location of forces to balance a meter stick.
- **Apparatus and Materials:**
 - 1 – 10g – 1000g Hooked Weight Set
 - 1 – Meter Stick
 - 1 – 1m Piece of String
 - 1 – Digital Force Gauge
- **Procedure:**

Part 1:

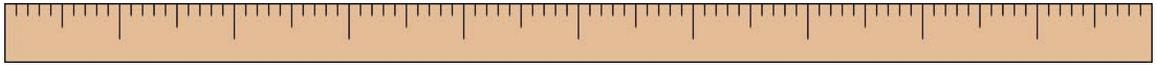
- First, the earth's gravity typically pulls a 1kg weight with a force of about 9.81N (N=Newton), but it varies a little depending where you are on the earth.
- Convert each weight value that you use to the earth's gravitational force.
- Verify each the calculated force by measuring the weight with the digital force gauge.
- Hang the meter stick from the overhead rail. Make a note of where the string is located on the meter stick when it is balanced. This is the center of gravity for the meter stick which you will need later in this lab.
- Take different two weights and use strings to hang both of the weights on the meter stick. Arrange the weights so that the meter stick is balanced. On the data sheet, draw the arrangement showing the forces created by the weights and their locations. Be sure to also show the location of your fulcrum in each of the diagrams.
- Calculate the torque from each of the weights and prove that they cause a net torque of zero. You will need to use the distance from the fulcrum (string holding the meter stick up) to each of the weights. Show all of your work on the data sheet. Make your torque units in newton-meters.

Part 2:

- Keep the balanced meter stick supported at the center of gravity.
- Choose two different weights.
- Pick one weight and hang the weight on the meter stick. Record the force from the weight and location of that weight on the data sheet.
- Knowing that the net torque on the meter stick is zero, calculate the location of the second weight on the data sheet.
- Place the weight at the calculated location. Is your system balanced? In a different color, show where you needed to move the weights to balance the system.

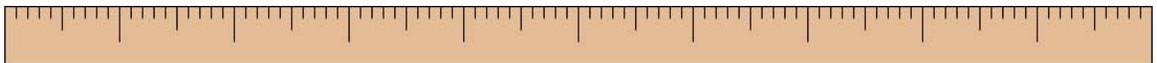
○ **Results:**

Part 1:



Length 1 (m)	Force 1 (N)	Torque 1 (Nm)
Length 2 (m)	Force 2 (N)	Torque 2 (Nm)

Part 2:



Length 1 (m)	Force 1 (N)	Torque 1 (Nm)
Length 2 (m)	Force 2 (N)	Torque 2 (Nm)